How Many Fixed Phase Shifters Are Needed in a Hybrid BF Structure?

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Outline

1. Introduction to beamforming
2. Hybrid beamforming by using fixed phase sifters
3. The feasible set
4. Simulation results
5. Conclusion

Higher spectral efficiency

In higher frequencies we can support larger bandwidths

Advantage

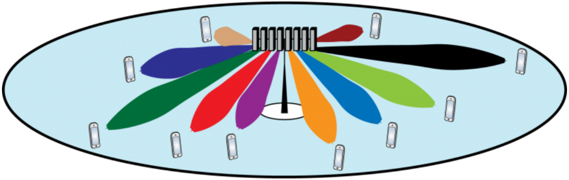
- Higher spectral efficiencies

One of the challenges

- The free space propagation loss is proportional to the square of the carrier frequency

The solution

- Generating narrower beams by using a large number of antennas

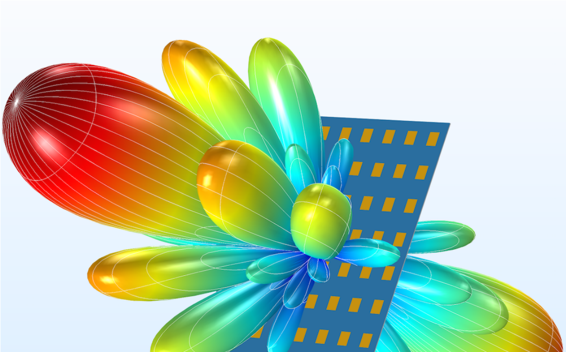


What is Beamforming?

The main goal of the beamforming is to direct the signal towards intended users

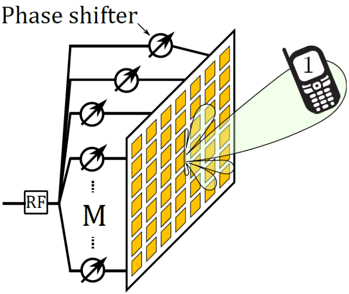
Beamforming can be performed as:

1. Full analog beamforming (AB)
2. Full digital beamforming (DB)
3. Hybrid beamforming (HB)



Full Analog beamforming

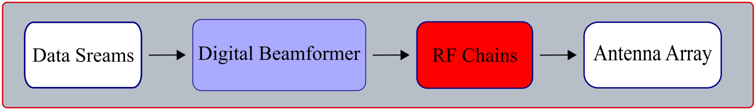
1. Requires only one RF chain
2. Needs one phase shifter per antenna
3. Performed in the RF domain
4. Low cost and complexity in implementation
5. Supports only one stream

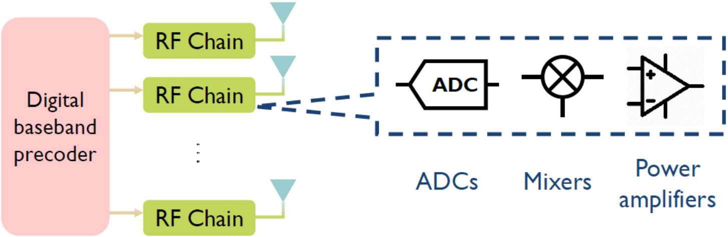


Full Digital Beamforming

1. Supports multi streams
2. Performed digitally at the baseband
3. Doesn't need any phase shifters
4. Requires an RF chain per antenna element
5. Costly and power hungry for massive MIMO systems

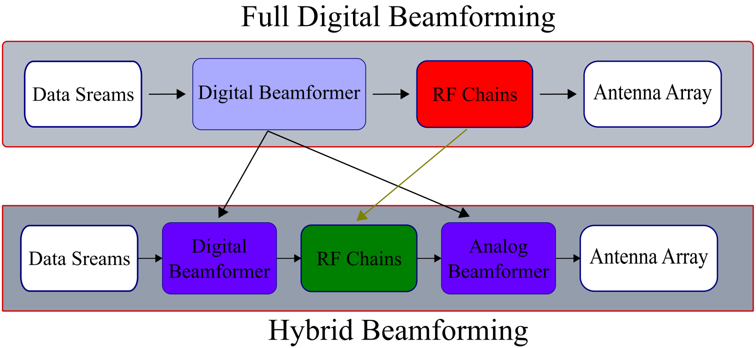
Full Digital Beamforming





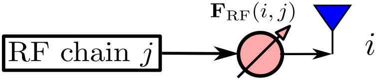
Hybrid Beamforming

1. Requires a few number of RF chains
2. Low power and cost (in comparison with full digital)
3. Needs digital and analog beamforming matrices
4. Requires higher computational complexity



How to perform the analog beamforming coefficients

Continuous PS



$$\mathbf{F}_{\mathrm{RF}}(i,j) = e^{\jmath\phi_{ij}}, \quad \phi_{ij} \in [0, 2\pi)$$

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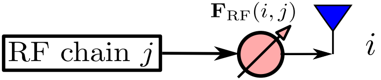

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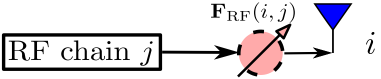
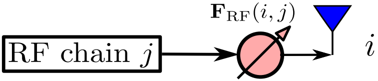
Quantized PS



$$\mathbf{F}_{\mathrm{RF}}\left(i,j\right) \in \left\{1, e^{\jmath 2\pi\frac{1}{N}}, \ldots, e^{\jmath 2\pi\frac{N-1}{N}}\right\},$$

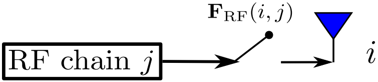
How to perform the analog beamforming coefficients

Continuous PS



$$\mathbf{F}_{\mathrm{RF}}(i,j) = e^{\jmath\phi_{ij}}, \quad \phi_{ij} \in [0, 2\pi)$$

- Switch



$$\mathbf{F}_{\mathrm{RF}}(i,j) \in \{0, 1\},$$

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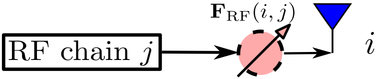

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How to perform the analog beamforming coefficients

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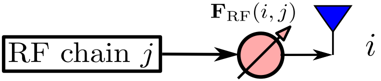

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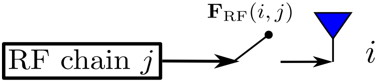

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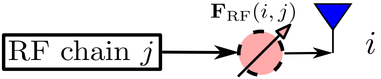

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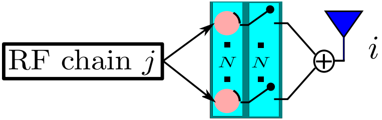
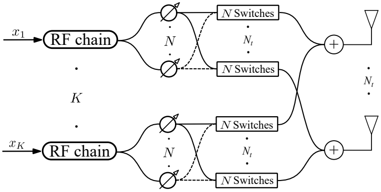
- Fixed PS with Switch



$$\mathbf{F}_{\mathrm{RF}}(i,j) \in \mathcal{F}_N,$$

System model deployed by FPSs and switches



$$\mathbf{y} = \sqrt{\rho\alpha}\mathbf{H}^{\mathsf{T}}\mathbf{F}_{\mathrm{RF}}\mathbf{x} + \mathbf{n}, \qquad (1)$$

- $\mathbf{y}$ is the received signal at users' side
- $\mathbf{x}$ is the data stream vector
- $\mathbf{n}$ is the noise vector
- $\mathbf{H}$ is the channel matrix
- $\mathbf{F}_{\mathrm{RF}}$ isthe analog BF matrix

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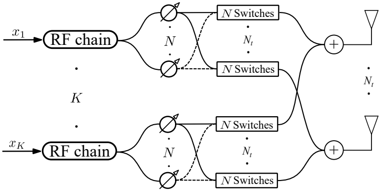

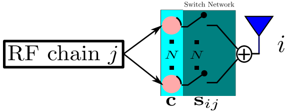

$$\mathbf{F}_{\mathrm{RF}}(i,j) = \mathbf{s}_{ij}\mathbf{c}$$

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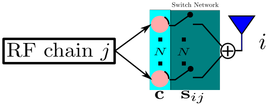
$$\mathbf{s}_{ij} = \underset{\mathbf{s}_{ij}}{\mathrm{argmin}} \quad |\gamma\mathbf{F}(i,j) - \mathbf{s}_{ij}\mathbf{c}|^2 \qquad (2)$$
$$\text{s.t.} \quad \mathbf{s}_{ij}(v) \in \{0,1\}, \quad \forall v = 1,\ldots,N.$$

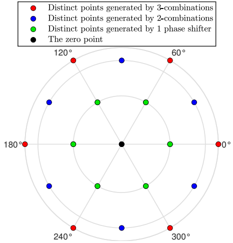
- γ is a scaling factor
- $\mathbf{F}$ is the fully digital precoder

The complete set $\mathcal{S}_N$



$\mathbf{F}_{\mathrm{RF}}(i, j) = \mathbf{s}_{ij}\mathbf{c}$

- $\mathbf{s}_{ij}$ is an $1 \times N$ binary vector.
- $\mathbf{c} = [P_1, P_2, \ldots, P_N]^{\mathrm{T}}$
- $P_m = e^{\jmath \frac{2\pi}{N}(m-1)}$



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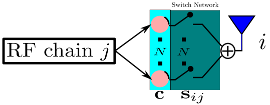

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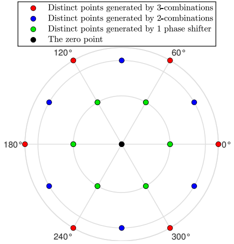
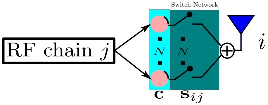


- $[N] = \{1, 2, ..., N\}$
- The power set $\Pi([N])$, is a set that contains all the subsets of the set $[N]$.
- The complete set $\mathcal{S}_N$, is a set that contains all generated coefficients:

$$\sum_{m \in \mathcal{A}_i} P_m \in \mathcal{S}_N \quad \forall i = 1, \dots, 2^N$$

- $\mathcal{A}_i$ is the ith member of the power set $\Pi([N])$

The complete set $\mathcal{S}_N$



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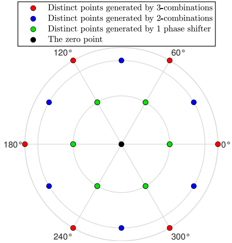

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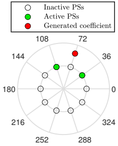
$$N = 10$$
$$\mathcal{A} = \{2, 4\}$$
$$\mathcal{S}_{10}(\mathcal{A}) = P_2 + P_4$$



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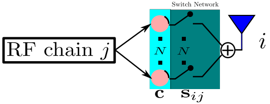

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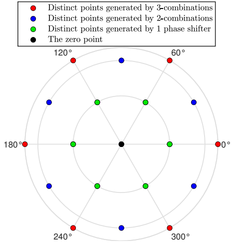

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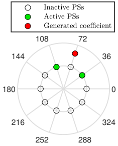

- Zero-summation set
- Subset of zero-summation set

Zero-Summation Set

Set $\mathcal{X}_{N,p}$ of size p (prime number), is a **zero-summation set** (ZSS) if:

$$\mathcal{S}_N\left(\mathcal{X}_{N,p}\right) = \sum_{m \in \mathcal{X}_{N,p}} P_m = 0$$

- This happens if and only if, $\mathcal{X}_{N,p}$ selects phase shifters with equal phase differences.
- Therefore, p is a prime factor of N, there are N/p different ZSSs, $\mathcal{X}^i_{N,p}$ $i = 1, \ldots, N/p$.

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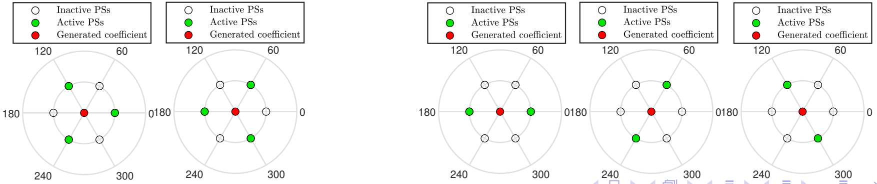
- If p is a prime factor of N, there are N/p different ZSSs, $\mathcal{X}_{N,p}^{i}$ $i = 1, \ldots, N/p$.

Ex: for $N = 6$, $p_1 = 3$, $p_2 = 2$

$\mathcal{X}_{6,3}^{1} = \{1, 3, 5\}$ $\mathcal{X}_{6,3}^{2} = \{2, 4, 6\}$ $\mathcal{X}_{6,2}^{1} = \{1, 4\}$ $\mathcal{X}_{6,2}^{2} = \{2, 5\}$ $\mathcal{X}_{6,2}^{3} = \{3, 6\}$



Zero-Summation Set

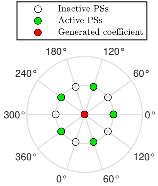
- If $\mathcal{A}_j = \mathcal{A}_i \cup \mathcal{X}_{N,p}$, $\mathcal{A}_j$ generates a superposed point onto $\mathcal{S}_N(\mathcal{A}_i)$ in $\mathcal{S}_N$, since:

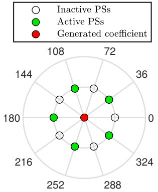
$$\mathcal{S}_N(\mathcal{A}_j) = \sum_{m \in \mathcal{A}_i} P_m + \sum_{m \in \mathcal{X}_{N,p}} P_m$$
$$= \sum_{m \in \mathcal{A}_i} P_m = \mathcal{S}_N(\mathcal{A}_i)$$

Subset of Zero-Summation Set

- Two phase shifters are dual of each other if the angle difference between them is 180-degree.
- Only for even N, there are dual phase shifters
- Consider $\mathcal{X}_{N,p}$ and its dual, $\bar{\mathcal{X}}_{N,p}$

$\mathcal{X}_{10,5} = \{1, 3, 5, 7, 9\}$





$\bar{\mathcal{X}}_{10,5} = \{2, 4, 6, 8, 10\}$

Subset of Zero-Summation Set

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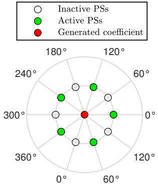
- Consider $\mathcal{X}_{N,p}$ and its dual, $\bar{\mathcal{X}}_{N,p}$

$$\mathcal{X}_{N,p} = \mathcal{R}_{N,p} \cup \mathcal{B}_{N,p}$$

$\mathcal{X}_{10,5} = \{1, 3, 5, 7, 9\}$



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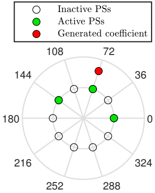
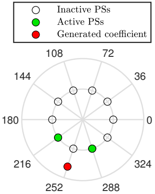
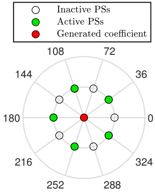
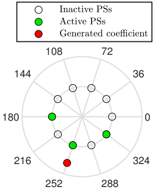


$\mathcal{B}_{10,5} = \{7, 9\}$

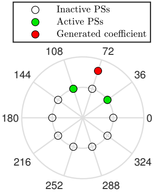




$\bar{\mathcal{X}}_{10,5} = \{2, 4, 6, 8, 10\}$



$\bar{\mathcal{R}}_{10,5} = \{6, 8, 10\}$



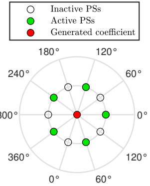
$\bar{\mathcal{B}}_{10,5} = \{2, 4\}$

Subset of Zero-Summation Set

- Two phase shifters are dual of each other if the angle difference between them is 180-degree.
- Only for even N, there are dual phase shifters
- Consider $\mathcal{X}_{N,p}$ and its dual, $\bar{\mathcal{X}}_{N,p}$

$$\mathcal{X}_{N,p} = \mathcal{R}_{N,p} \cup \mathcal{B}_{N,p}$$

$\mathcal{X}_{10,5} = \{1, 3, 5, 7, 9\}$



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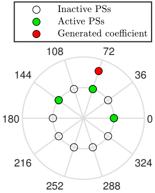

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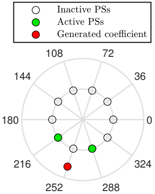

$$\sum_{m \in \mathcal{B}_{N,p}} P_m + \sum_{m \in \mathcal{R}_{N,p}} P_m = 0$$

$$\sum_{m \in \mathcal{R}_{N,p}} P_m + \sum_{m \in \bar{\mathcal{R}}_{N,p}} P_m = 0$$

$$\sum_{m \in \mathcal{R}_{N,p}} P_m = \sum_{m \in \bar{\mathcal{B}}_{N,p}} P_m$$

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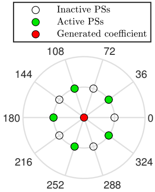
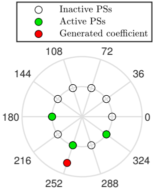
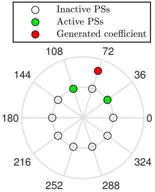


$\bar{\mathcal{X}}_{10,5} = \{2, 4, 6, 8, 10\}$



$\bar{\mathcal{R}}_{10,5} = \{6, 8, 10\}$



$\bar{\mathcal{B}}_{10,5} = \{2, 4\}$

Subset of Zero-Summation Set

$$\sum_{m \in \mathcal{R}_{N,p}} P_m = \sum_{m \in \bar{\mathcal{B}}_{N,p}} P_m$$

For two sets $\mathcal{A}_i$ and $\mathcal{A}_j$ with common subset $\mathcal{A}_c = \mathcal{A}_i \cap \mathcal{A}_j$, if

$$\begin{cases} \mathcal{A}_i \cap \mathcal{X}_{N,p} = \mathcal{R}_{N,p} \\ \mathcal{A}_i \cap \bar{\mathcal{X}}_{N,p} = \{\}\,, \end{cases} \quad , \quad \begin{cases} \mathcal{A}_j \cap \mathcal{X}_{N,p} = \{\} \\ \mathcal{A}_j \cap \bar{\mathcal{X}}_{N,p} = \bar{\mathcal{B}}_{N,p}, \end{cases}$$

$\mathcal{A}_i$ generates a superposed point of $\mathcal{S}_N(\mathcal{A}_j)$, since:

$$\begin{aligned} \mathcal{S}_N(\mathcal{A}_i) &= \sum_{m \in \mathcal{A}_c} P_m + \sum_{m \in \mathcal{R}_{N,p}} P_m \\ &= \sum_{m \in \mathcal{A}_c} P_m + \sum_{m \in \bar{\mathcal{B}}_{N,p}} P_m \\ &= \mathcal{S}_N(\mathcal{A}_j) \end{aligned}$$

For $p = 2$, there is no SZSS since $\mathcal{X}_{N,2} = \bar{\mathcal{X}}_{N,2}$

The size of the feasible set $\mathcal{F}_N$

The size of the feasible set $|\mathcal{F}_N|$ as:

$$|\mathcal{F}_N| = \sum_{r=0}^{\xi_N - 1} \binom{N}{r} - \mathcal{Z}_N(r) - \mathcal{Q}_N(r) \tag{3}$$

- $\mathcal{Z}_N(r)$ represents the number of members of $\Pi_r([N])$ containing at least one ZSS
- $\mathcal{Q}_N(r)$ represents the number of members of $\Pi_r([N])$ containing at least one SZSS.
- ξ_N active phase shifters create at least one $\mathcal{X}_{N,p_1}$, where $\xi_N = N - \frac{N}{p_1} + 1$.

The obtained analytical results

N	$\lvert\mathcal{S}_N\rvert$	$\lvert\mathcal{F}_N\rvert$	$\frac{\lvert\mathcal{F}_N\rvert}{\lvert\mathcal{S}_N\rvert}$
2	4	3	75.00%
3	8	7	87.50%
4	16	9	56.25%
5	32	31	96.88%
6	64	19	29.69%
7	128	127	99.22%
8	256	81	31.64%
9	512	343	66.99%
10	1024	211	20.61%
11	2048	2047	99.95%
12	4096	361	08.81%
13	8192	8191	99.99%
14	16384	2059	12.57%
15	32768	16081	49.08%
16	65536	6561	10.01%
17	131072	131071	100.00%

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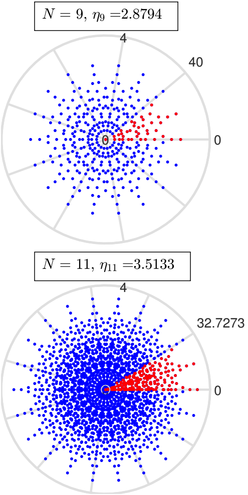
Scatter plot of the feasible set



Spectral Efficiency

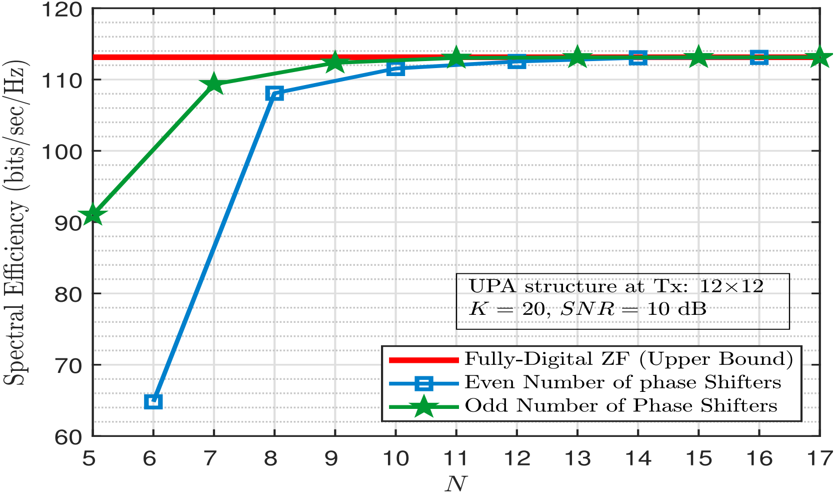


Figure: Performance comparison for different number of phase shifters, N.

Spectral Efficiency

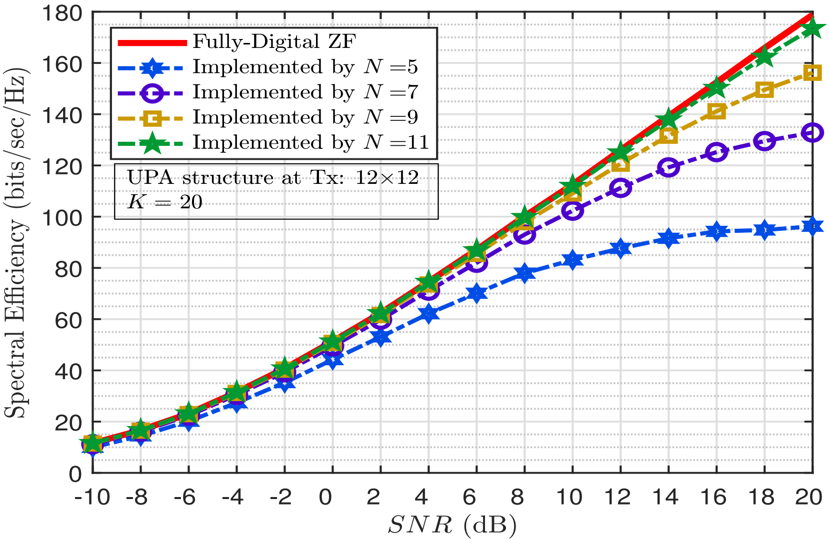


Figure: Performance comparison for different values of SNR

Conclusion

- Performance depends on the number of distinct coefficients
- An even number of PSs is not a good choice at all
- A prime number of PSs is the most efficient selection
- 11 PSs is a good choice

Thanks For Your Attention